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Combining optimal guidance law and differential game guidance law to improve the guidance efficiency

Nguyen Minh Hong

Problem statement and specific task. *When a missile is aimed at a target according to the traditional law, normal acceleration of the rocket at the meeting point of the target is very large and exceeds the permissible value of the missile. The article deals with the task of building a hybrid rocket guidance law to overcome some of the disadvantages of traditional rules and to ensure a small slip.* ***Purpose of the work****: The article develops a hybrid law of guidance, which based on a combination of the theory of optimal control and the guidance law of the differential game, to improve guidance accuracy when the target moves according to different rules.* ***The results*** *of the study showed that when applying the proposed hybrid law of guidance to the guidance system for the maneuver objective, the root-mean-square slippage is quite small as when applying the law of guidance for the differential game.* ***Scientific and practical value:*** *The results of practical research and proposed methods for the synthesis of the guidance law can be used to improve the accuracy of the guidance system in self-guide air-to-air missiles and surface-to-air missiles. through simulation results.*

Key words: *guidance law, differential game, guidance efficiency, target, normal acceleration.*

1. Introduction

Today, with the development of science and technology, the flying equipment in general have been developed ceaselessly in all aspects. Thanks to the application of achievements of new materials, they are able to withstand collisions at a certain level. Therefore, the class of traditional missiles uses splinters coming off warheads to destroy targets that are difficult to be shot down completely [1, 2]. It then requires the missiles to hit directly to destroy the target or the slippage to be very small. However, when applying traditional guidance laws, in order to achieve the small slippage, the missile`s normal acceleration at the time of collision must be very large and exceed the missile's permitted overload [3]. In addition, the maneuverability of the target has been improved ceaselessly in terms of the permitted overload as well as the type of maneuverability.

Coming from these two reasons, a great number of studies have been published to design a new guidance law for missiles to ensure the minimal slippage and ensure that the missile does not have to maneuver too much in comparison with the target`s maneuverability. Among the achieved results, the optimal guidance law (OGL is one of the guidance law that can meet the above requirements by putting the square parameters of normal acceleration command into the cost function to solve the

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problem of optimization with the constraints of slippage at the meeting time [3]. However, the main disadvantage of the OGL is the weak stability because the OGL require the accurate modeling of missile dynamics and information about the target's normal acceleration. In fact, these information are difficult to be identified accurately, then the quality of the OGL rule will be considerably reduced [4].

Because the maneuverability strategy of the target is independent and uncontrolled, or in other words unpredictable, some authors consider the problem of missile guidance as a chase-run game with the zero total between the missile (chase) and target (run), and the proposed guidance law that is based on this idea is called as the Differential Game (DG) guidance law [5, 6, 7]. Similar to the OGL, the DG guidance law is also obtained by solving the problem of optimal control in the linear quadratic form. However, the OGL is obtained by solving the one-sided optimization problem, ie, the OGL assumed that the target's maneuverability strategy has been known [6]. Whereas the DG guidance law is obtained by solving the two-sided optimization problem, that is, the missile and the target are considered as two independently controlled objects. In which, the missile always tries to minimize the slippage (to catch up with the target), while the target always tries to maneuver to maximize the slippage (to escape from the missile) [7]. One of the advantages of the DG guidance law is that it does not use information of the target`s maneuverability, but still ensure the slippage to be small at the meeting time [7]. However, this guidance law has not been applied in reality because of the sudden change in the missile`s normal acceleration command [8, 9] (the normal acceleration will reach or exceed the negative or positive permitted overload) which significantly increases the aerodynamic drag force impacted on the missile.

Therefore, this article will propose a solution to overcome the disadvantages and exploit the advantages of the OGL and DG guidance law by applying both of these guidance laws during the flying process of the missile. First, the article will survey and compare the quality of the two guidance laws on the same with the proportional navigation (PN) guidance law when the self-guided loop is affected by interference, the fact that the target maneuvers in two different manners and the fact that the missile`s maneuverability is limited in comparison with the target`s. Based on the obtained results, the combined guidance law will be proposed to achieve the best guidance quality.

2. Guidance problem surveying model

First, the article considers the dynamic geometric relation between the missile and the target in the horizontal plane as shown in Figure 1 [10].

With the assumption that the flight angle of the missile λ and the target is β small enough, we have the following equations [10]:

$$\ddot{y} = n_T \cos\beta - n_c cos\lambda \approx n_T - n_c \ , \quad (1)$$

$$\lambda \approx \sin\lambda = \frac{y}{R_{TM}} \ . \quad (2)$$

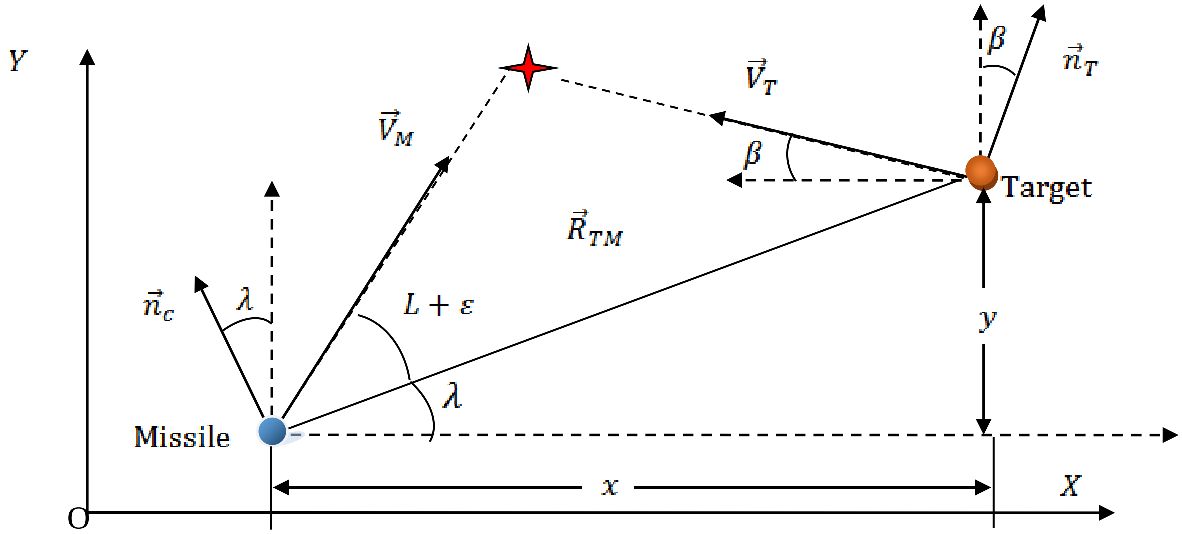

Fig. 1. Dynamic geometric relation between the missile and the target in the vertical plane

We then map the system structure as shown in Figure 2.

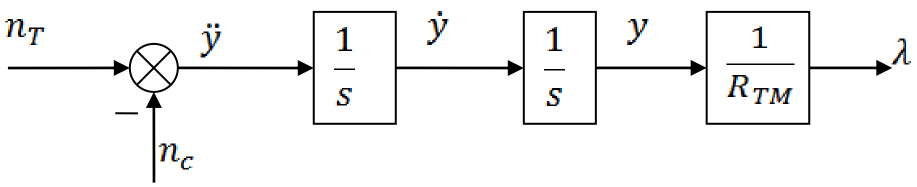

Fig. 2. Diagram of the ideal guidance system structure

In order to survey the impacts of the interference, type of maneuverability of the target and the maneuverability of the missile compared to the target`s maneuverability to the guidance system when applying different guidance laws, the article uses the self-guided loop as shown in Figure 3 below:

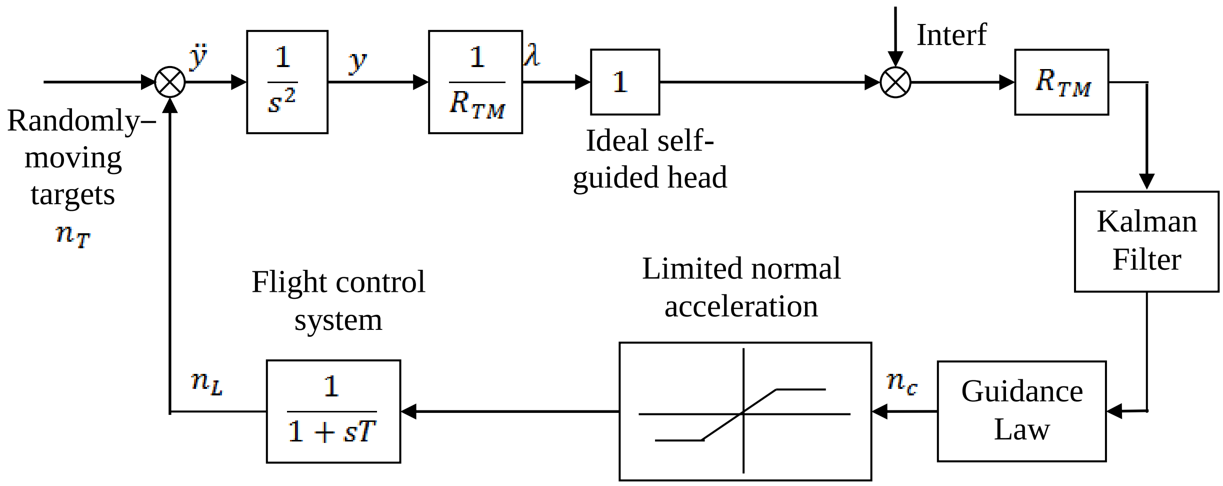

Fig. 3. Self-guided guidance law survey loop

In which:

1) Target maneuvers according to two laws as follows:

- Maneuver with a constant normal acceleration n_T , sometimes maneuver randomly, uniformly distribute during the whole flight duration of the missile.
- Maneuver according to the random telegraph signal (Poisson square pulses) with the amplitude n_T and frequency k times of changing the sign $P(k)$ in the period t_F determined by the Poisson distribution as follows [10]:

$$P(k)=\frac{(\vartheta t_F)^k e^{-\vartheta t_F}}{k!}, \tag{3}$$

with ϑ the average number of sign changing times in one second.

2) Self-guided head is considered to be ideal.

3) Interference in the angle measurement with the standard deviation of 0.1 mrad.

4) Use the three-state Kalman filter to estimate information about the distance, y, speed $\dot{y}$ and normal acceleration of the target n_T [11]. The sampling rate of the filter is 0.01s.

5) The permitted normal acceleration of the missile: $n_{c_{max}}$

6) The flight control system is modeled as a first-grade inertia stage with the time constant $T=0.2s$.

3. Summary of combined guidance law

The article will evaluate the influences of random factors on the guidance system according to the Monte Carlo method [12]. Three guidance laws that are surveyed to evaluate the influences of random factors on the quality of the guidance system are:

- The proportional navigation guidance law with the expression defining the normal acceleration as follows [3]:

$$n_{cPN}=N'V_c\dot{\lambda} \tag{4}$$

In which, $N'$is a guidance constant, receiving the integer value from 3 to 5, V_c is the approach velocity, and $\dot{\lambda}$ is the angular velocity of sight line.

- The optimal guidance law [3] has the expression determining the optimal normal acceleration command when the flight control system is modeled as a first-grade inertia stage with a time constant of T in the form:

$$n_{cOGL}=\frac{N'}{t_{go}^2}\left[y+\dot{y}t_{go}+0.5n_Tt_{go}^2-n_LT^2\left(e^{-x}+x-1\right)\right], \tag{5}$$

with

$$N'=\frac{6x^2\left(e^{-x}-1+x\right)}{2x^3+3+6x-6x^2-12xe^{-x}-3e^{-2x}}.$$

Differential game guidance law with the normal acceleration command in the form:

$$n_{cDG}=n_{c_{max}}sign\left[y+\dot{y}t_{go}-n_LT^2\left(e^{-x}+x-1\right)\right]. \tag{6}$$

In which, $sign(.)$ is the sign function. In expressions (5) and (6), t_{go} and x are defined as follows:

$$t_{go} = t_F - t, \;\; x = \frac{t_{go}}{T},$$

t_F is the meeting time between the missile and the target.

For each type of maneuverability of the target, the article will simultaneously survey all of the three laws at the meeting time $0s < t_F < 10s$. At each value t_F, the article carried out 50 Monte Carlo simulations to determine the root-mean-square slippage at the meeting time.

The following is the survey results when the target maneuvered according to three different laws in turn with the amplitude $5g$ and the missile`s permitted normal acceleration of $10g$.

In case the target moves with a constant amplitude

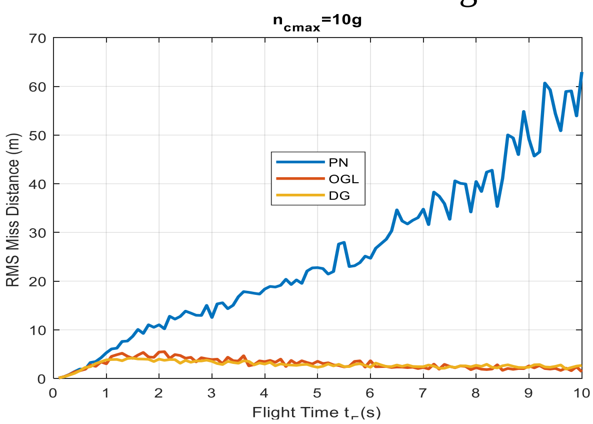

Fig. 4. Root-mean-square slippage

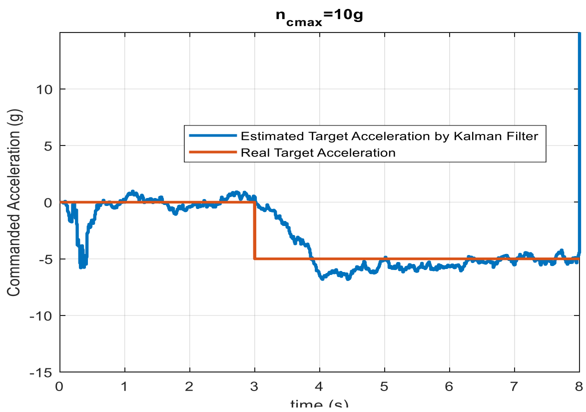

Fig. 5. Estimated target normal acceleration when $t_F = 8s$

In case the target moves in the form of random telegraph signal

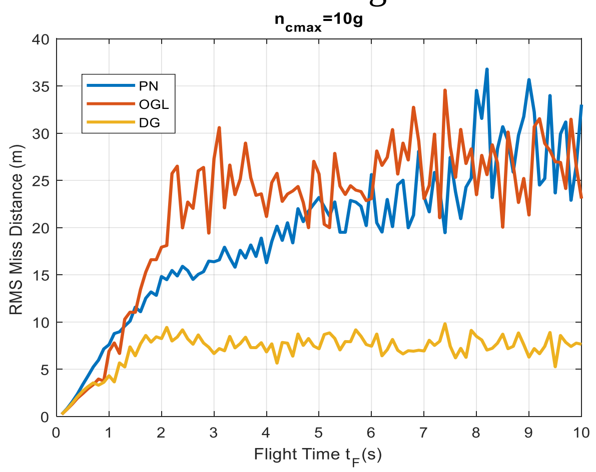

Fig. 6. Root-mean-square slippage

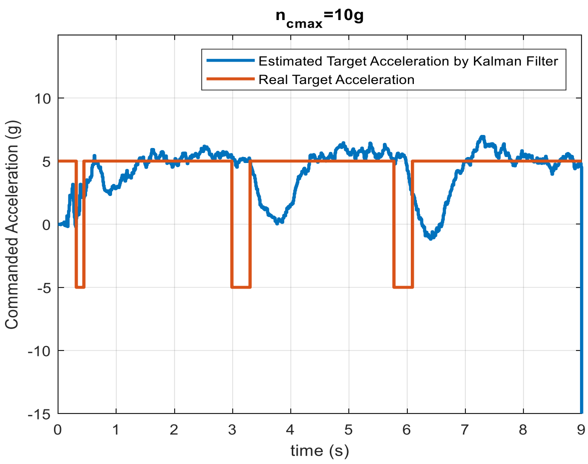

Fig. 7. Estimated normal acceleration of the target when $t_F = 9s$

Fig. 4 shows that the permitted maneuverability of the missile is 10g (as twice as the normal acceleration of the target), the slippage increases rapidly when the PN guidance law is applied, which indicates that when the maneuverability of the missile is limited, the PN guidance law is no longer appropriate. Fig. 5 depicts the normal acceleration value estimated by the Kalman filter; it can be seen that the Kalman filter has accurately estimated the target normal acceleration. However, when the maneuverability law of the target is more complex (the type of random telegraph signal), the Kalman filter is almost impossible to guarantee the accuracy of the estimated normal acceleration of the target (Fig. 7). It`s the inaccuracy in the estimation of the target normal acceleration, the quality of the OGL has been considerably reduced (Fig. 6). However, if viewed as a whole, when the system is affected by the interference and the limited maneuverability of the missile, the OGL still brings better quality than the PN guidance law.

In both cases of the target`s maneuverability, the DG guidance law always brings the best quality, the quality of the DG guidance law does not depend on the maneuverability type of the target like the OGL. However, the biggest disadvantage of the DG guidance law is the sudden change in the normal deceleration command (always equal to the maximum value of the permitted normal acceleration command of the missile). The following is an example illustrating the normal deceleration command when applying the DG guidance law along with the PN guidance law and the OGL when the target moves at a constant amplitude of 5g and starts moving at the 3rd second.

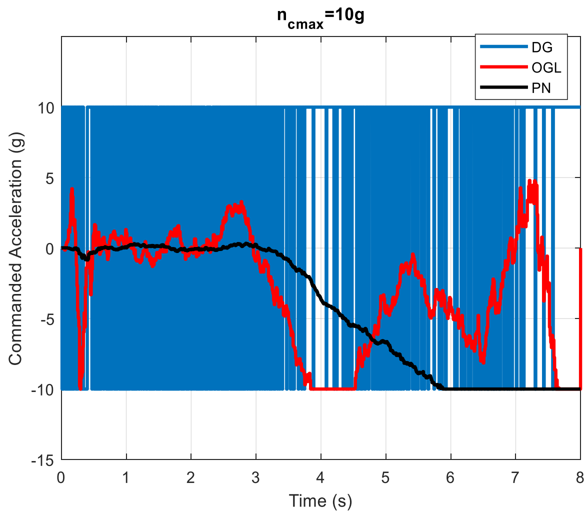

Fig. 8. Missile normal acceleration command

Based on the above comments, the article proposes a hybrid guidance (HG) law combining the OGL and the DG guidance law as follows:

$$n_{cHG} = \begin{cases} n_{cOGL}, & if\ t_{go} > t_{change}; \\ n_{cDG}, & if\ t_{go} \le t_{change}. \end{cases} \tag{7}$$

In which n_{cOGL} is the missile`s normal acceleration command when applying the OGL and defined by (5); $n_{DG}$ is the missile`s normal acceleration command when applying the DG guidance law and defined according to (6); t_{change} is the duration since when converting between the two guidance laws up to the meeting point.

4. Survey results

In case the target moves with a constant amplitude

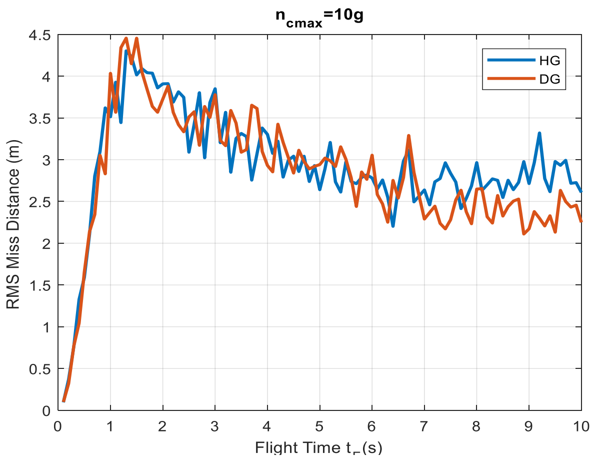

Fig. 9. Root-mean-square slippage when $t_{change} = 1s$

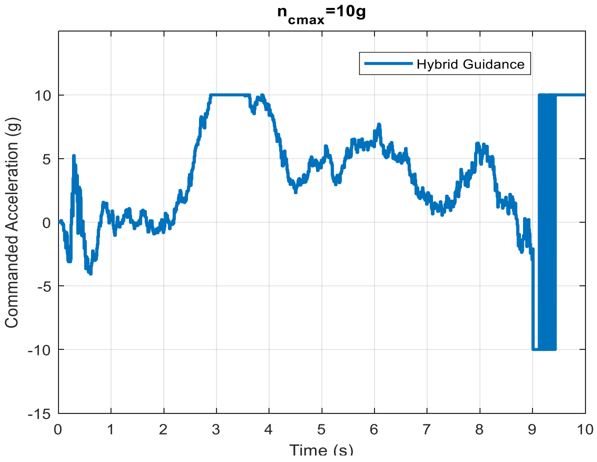

Fig. 10. A case of the missile`s normal acceleration with *1s* $t_F = 10s$ and $t_{change} = 1s$

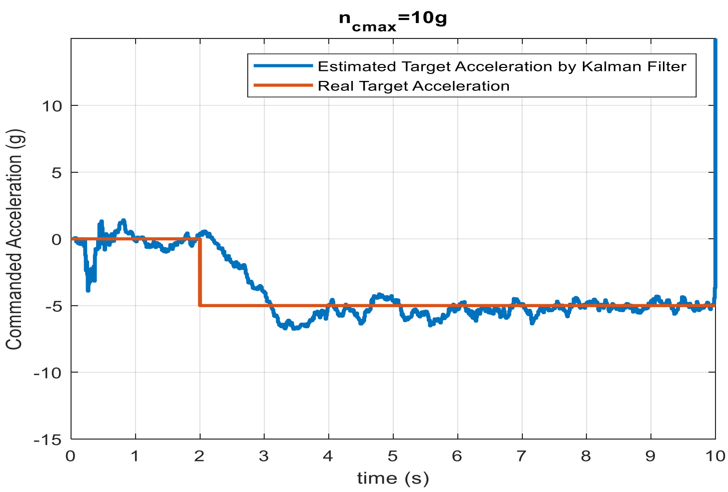

Fig. 11. Estimated normal acceleration of the target when $t_F = 10s$

In case the target moves in the form of random telegraph signal

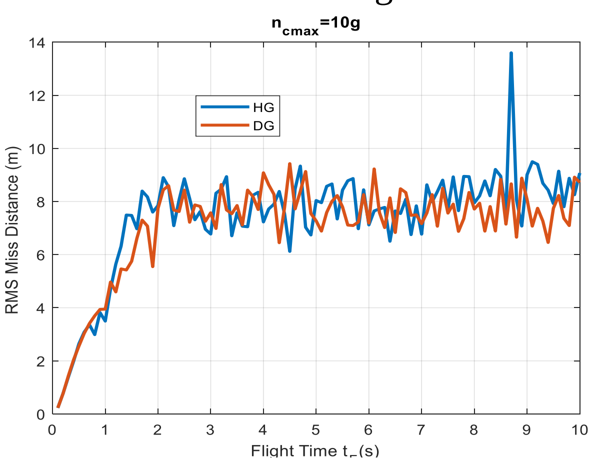

Fig. 12. Root-mean-square slippage when $t_{change} = 2s$

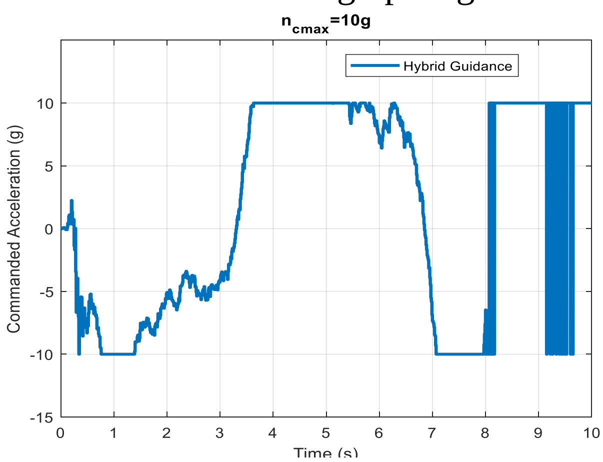

Fig. 13. A case of the missile`s normal acceleration with *1s* $t_F = 10s$ and $t_{change} = 2s$

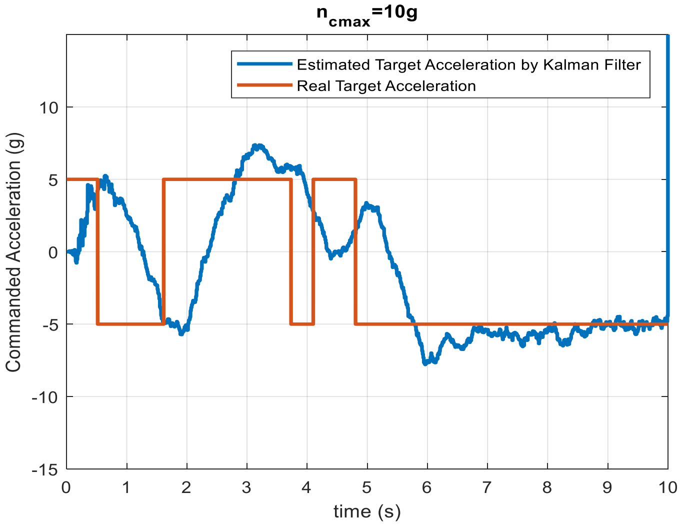

Fig. 14. Estimated normal acceleration of the target when $t_F = 10s$

Fig. 9 shows the lines presenting the root-mean-square slippage when applying the HG law and the DG guidance law that are quite similar to each other even though the DG guidance law is applied in the last 1 second of the flight process. This result shows that the DG guidance law plays an important role in determining the guidance quality. However, when surveying the HG guidance law with the target moving in the form of random telegraph signal, because the Kalman filter is no longer able to accurately estimates the target normal acceleration (Fig. 14) as in the case of a fixed maneuverability of the target (Fig. 11), the quality in the initial phase (using the OGL) has been considerably reduced. In order to ensure the guidance quality when using the HG law to be similar to the DG guidance law (Fig. 12), the article increased the period of applying the DG guidance law to 2 seconds (calculated from the time of application to the meeting time)

By combining these two guidance laws, the number of sudden changes in the missile's normal acceleration has been reduced to about 80% as shown in Fig. 10 and Fig. 13.

5. Conclusion

This article has surveyed 3 guidance laws: PN, OGL and DG to analyze the advantages as well as disadvantages of each guidance law. On that basis, it is proposed to combine the OGL and the DG guidance law in order to overcome the limitations and exploit the strengths of each guidance law. The survey results showed that when applying the proposed hybrid guidance law into the guidance system affected by the interference in the angle measurement and the maneuverable target according to the two laws with different degrees of difficulty, the root-mean-square slippage is still ensured to be as small as when applying the DG guidance law. In addition, the article significantly reduced the sudden changes in the missile`s normal acceleration command received by applying the proposed guidance law.

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Эффективное наведение ракеты на цель на основе интеграции оптимального управления и дифференциальной игры

Нгуен М. Х.

Постановки задачи: *При наведении ракеты на цель в режиме оптимального управления, ускорение ракеты в точке встречи с целью достаточно большое и превышает значение, являющееся допустимым ракеты. В статье рассматривается задача формирования гибридного режима управления наведением ракеты, который позволяет преодолеть указанные недостатки при наведении ракеты к режиме оптимального управления и минимизировать промах.* ***Цель работы:*** *формирование гибридного режима наведения на основе соединения режима наведения по теории оптимального управления с учетом формализации процесса наведения как дифференциальной игры.* ***Результаты*** *исследования показали, что при внедрении предлагаемого гибридного режима наведения в систему наведения ракетой* *обеспечивается более низкое значение ее промаха по показателю среднеквадратичного отклонения наведения на цель.* ***Научная и практическая значимости:*** *результаты практического исследования и предлагаемый режим гибридного наведения могут использоваться с целью повышения точности самонаводящихся ракет класса «воздух-воздух» и «земля-воздух».*

Ключевые слова: *эффективность, оптимальный закон наведения, ракета, дифференциальная игра, маневренность.*

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